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Adaptive sliding control of non-autonomous active suspension systems with time-varying loadings

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Abstract

An adaptive sliding controller is proposed in this paper for controlling a non-autonomous quarter-car suspension system with time-varying loadings. The bound of the car-body loading is assumed to be available. Then, the reference coordinate is placed at the static position under the nominal loading so that the system dynamic equation is derived. Due to spring nonlinearities, the system property becomes asymmetric after coordinate transformation. Besides, in practical cases, system parameters are not easy to be obtained precisely for controller design. Therefore, in this paper, system uncertainties are lumped into two unknown time-varying functions. Since the variation bound of one of the unknown functions is not available, conventional adaptive schemes and robust designs are not applicable. To deal with this problem, the function approximation technique is employed to represent the unknown function as a finite combination of basis functions. The Lyapunov direct method can thus be used to find adaptive laws for updating coefficients in the approximating series and to prove stability of the closed-loop system. Since the position and velocity measurements of the unsprung mass are lumped into the unknown function, there is no need to install sensors on the axle and wheel assembly in the actual implementation. Simulation results are presented to show the performance of the proposed strategy.

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1. Introduction

In recent years, various control methods on active suspension systems have been developed to improve ride comfort and handling quality under the constraints of realizable control force and feasible motion trajectories of vehicle operations [2]. Since the suspension might exhibit large variation in system states under severe driving conditions, system nonlinearities should be considered to maintain the ride quality. On the other hand, because the loading conditions may vary and the suspension parameters may change, all kinds of uncertainties should be investigated.

To deal with the problem stated above, many researches [1–12] have been proposed. Yoshimura et al. [1] used the concept of sliding mode control to construct a pneumatic suspension system and the unknown road profile was estimated by using a minimum-order observer. The nonlinearities of passive components and the tire lift-off phenomenon were considered in Kim et al. [3], and a sliding mode controller was designed to give desired tracking performance of the motion trajectories to a reference sky-hook damping system. Tuan et al. [4] proposed a nonlinear H_∞ controller based on the parameterized LMI approach for an integrated active suspension. Karlsson et al. [5,6] designed a nonlinear H_∞ controller to minimize the L_2 gain from the road disturbance to a quadratic cost function. Sunwoo et al. [7] suggested a model reference adaptive controller for suspension systems with acceleration feedback at the wheel hub. Nonlinear adaptive controllers were designed by Alleyne and Hedrick [8] for the force tracking of electrohydraulic actuators to a desired sky-hook damping dynamics. Along the same way as in Alleyne and Hedrick [8], Chen and Huang [12] proposed a function approximation-based adaptive sliding controller for a hydraulic actuator to give satisfactory force tracking performance regardless of time-varying uncertainties in the actuator. Kim [9] developed a nonlinear indirect adaptive controller for a reduced-order suspension system. Some other hybrid control strategies were developed in [10,11] to deal with both the dynamics of the car-body part and the uncertainties in the actuator.

In this paper, we would like to propose an adaptive sliding controller for a non-autonomous quarter-car suspension system with bounded time-varying loadings. The system dynamic equations are firstly derived with respect to the static positions under the car-body nominal load with the assumption that the variation bound of the car-body load is available. Due to the nominal load, the nonlinear spring properties become asymmetric. Since the load might be large, there is significant discrepancy of the behaviors between the nonlinear spring and its linearized model. Hence, the spring nonlinearities should be well compensated to maintain the control performance for a wide range of vehicle operations. However, it is quite difficult to obtain precise system parameters, since the practical system is inherently nonlinear and uncertain. Therefore, in this paper, all parameters in system model are assumed to be unavailable except that the variation bound of car-body load is known. With the assumptions, the system is further represented as a second-order input-to-output dynamics with a stable internal dynamics and the system uncertainties as well as the internal states are lumped into an unknown time-varying function with an unknown bound. If the lumped uncertainty has been properly compensated, there is no need to feedback these internal states in actual implementation. Because the uncertainties are time variant, conventional adaptive schemes are not applicable. On the other hand, since the variation bounds of the uncertainties are not given, most robust designs fail. Here, the function approximation approach [12–15] is employed. The basic idea is to represent the system uncertainties using finite linear combinations of basis functions with unknown constant weighting vectors. Output error dynamics can thus be derived as a stable first-order filter driven by parameter error vectors. Appropriate update laws for the weighting vectors can be selected based on the Lyapunov stability theory. Asymptotic stability of the output error can be obtained if sufficient number of basis functions is used. Effects of the approximation error on system performance are also investigated in this paper. It is proved that, for bounded approximation errors, the output error is ultimately bounded. If, in addition, the bounds of approximation errors are known, asymptotic convergence of the output error still can be obtained with a modified control law.

This paper is organized as follows: Section 2 gives the problem formulation. Section 3 reviews the function approximation technique. An adaptive sliding controller is designed in Section 4 with rigorous proof of closed-loop stability. Section 5 presents results of computer simulations of the proposed controller. Section 6 concludes this paper.

2. Problem formulation

A well-known quarter-car model of the suspension system is shown in Fig. 1. The sprung mass $m_s(t)$ represents the time-varying mass of the car-body part and the unsprung mass m_u is the assembly of the axle and wheel. The tire is modeled as a combination of a linear spring and damper with coefficients k_t and c_t, respectively. The time-varying damper $c_s(t)$ and nonlinear spring k_s comprise the passive components of the suspension system. The actuator force u is acting between the sprung and unsprung masses, whose magnitude is bounded by some known values, i.e., $|u| \leqslant u_{\max}$ for some $u_{\max} > 0$. The variable z is the uneven road input to the unsprung mass dynamics.

Let x_s and x_u be the vertical displacements of the sprung and unsprung masses with respect to the undeformed suspension positions, respectively. The dynamic equations of the suspension

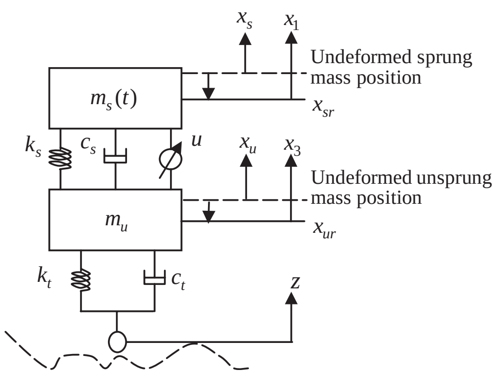

Fig. 1. Quarter car suspension model.

system can be expressed as

$$m_s(t)\ddot{x}_s = -k_{1s}(x_s - x_u) - k_{2s}(x_s - x_u)^3 - c_s(t)(\dot{x}_s - \dot{x}_u) + u - m_s(t)g, \tag{1a}$$

$$m_u\ddot{x}_u = k_{1s}(x_s - x_u) + k_{2s}(x_s - x_u)^3 + c_s(t)(\dot{x}_s - \dot{x}_u) - k_t(x_u - z) - c_t(\dot{x}_u - \dot{z}) - u - m_u g, \tag{1b}$$

where k_{1s} and k_{2s} are, respectively, linear and nonlinear stiffness coefficients of the spring k_s. Since the variation range of car-body loads is easy to be estimated in practical applications, it is reasonable to assume that the bound of $m_s(t)$ is known, and hence $m_s(t)$ can be further represented as $m_s(t) = m_{sm} + \Delta m_s(t)$ where the positive constant m_{sm} is the nominal value and $\Delta m_s(t)$ is the additive uncertainty with a known bound. Define the reference position of the unsprung mass as $x_{ur} = -(m_u + m_{sm})g/k_t$ which is exactly the static tire deflection due to the nominal load $m_{sm}g$. Likewise, we may define the reference position of the sprung mass as $x_{sr} = x_{ur} + \delta_0$ where $\delta_0 < 0$ is the static spring deflection. Let the state variables be $x_1 = x_s - x_{sr}$, $x_2 = \dot{x}_s$, $x_3 = x_u - x_{ur}$ and $x_4 = \dot{x}_u$, then dynamic equation (1) can be rewritten as

$$\dot{x}_1 = x_2, \tag{2a}$$

$$\dot{x}_2 = \frac{1}{m_s(t)}[-k_{1s}(x_1 - x_3) - k_{2s}\phi(x_1, x_3) - c_s(t)(x_2 - x_4) - \Delta m_s(t)g + u], \tag{2b}$$

$$\dot{x}_3 = x_4, \tag{2c}$$

$$\dot{x}_4 = \frac{1}{m_u}[k_{1s}(x_1 - x_3) + k_{2s}\phi(x_1, x_3) + c_s(t)(x_2 - x_4) - u - k_t(x_3 - z) - c_t(x_4 - \dot{z})], \tag{2d}$$

where $\phi(x_1, x_3) = (x_1 - x_3 + \delta_0)^3 - \delta_0^3$ is the spring nonlinearity. To further investigate the effect of the nonlinear spring, the spring force $k_{1s}(x_s - x_u) + k_{2s}(x_s - x_u)^3$ considered in Eq. (1) is depicted as the solid curve in Fig. 2. Due to the nominal load $m_{sm}g$, the reference position of the

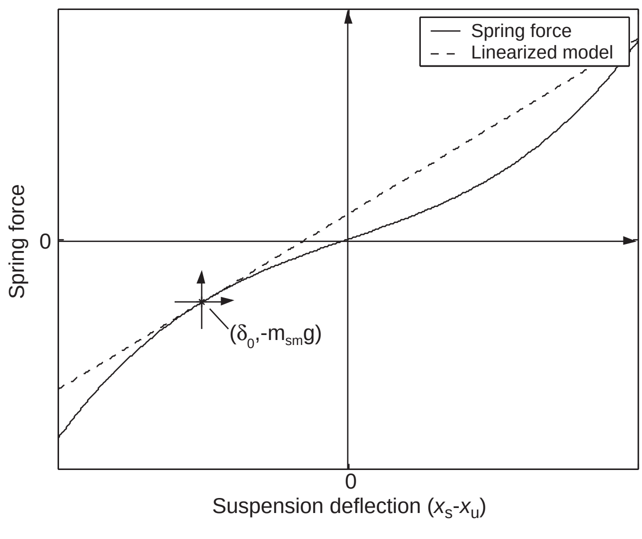

Fig. 2. Nonlinear spring properties.

spring moves from the origin to the point $(\delta_0, -m_{sm}g)$. This might result in significant discrepancy of the behaviors between the nonlinear spring and its linearized model (dash line), if the nominal load is large. Hence, to have better control performance for a wide range of the car-body loading, we have to consider the effect of spring nonlinearity. Furthermore, since the damper viscosity may vary as the fluid characteristics change, the damping coefficient $c_s(t)$ is considered to be time-varying.

Due to inherent nonlinearities and uncertainties in practical systems, precise parameters may not be easy to be obtained for controller design. Hence, in this paper, all parameters in (2) are assumed to be unavailable, except that the variation bound of $m_s(t)$ is known. The objective is to find a controller so that all signals in the system remain bounded and the car-body displacement x_1 converges to zero asymptotically. Besides, to simplify its implementation, only x_1 and x_2 are measured, and hence there is no need to install sensors on the wheel and axle assembly. To this end, we may define $y = x_1$ as the system output, and the relative degree from the input to the output is apparently two. This input-to-output dynamics can be represented as

$$\ddot{y} = f(\mathbf{x}, t) + g(t)u, \tag{3}$$

where $\mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^{\mathrm{T}}$ is the state vector and $f(\mathbf{x}, t)$ and $g(t)$ are, respectively, in the forms

$$f(\mathbf{x}, t) = \frac{1}{m_s(t)}[-k_{1s}(x_1 - x_3) + k_{2s}\phi(x_1, x_3) - c_s(t)(x_2 - x_4) - \Delta m_s(t)g], \tag{4a}$$

$$g(t) = \frac{1}{m_s(t)}. \tag{4b}$$

Since most quantities in $f(\mathbf{x}, t)$ are unavailable, we have to regard $f(\mathbf{x}, t)$ as an unknown time-varying function whose variation bound is not known either. On the other hand, since an estimate of the bound of $m_s(t)$ is possible, the unknown function $g(t)$ has known bounds, i.e., $g_{\min} \leqslant g(t) \leqslant g_{\max}$ with some known $g_{\min}$ and $g_{\max}$. Suppose $g(t)$ is modeled as $g(t) = g_m \Delta g$, where g_m is the nominal value and Δg is the multiplicative uncertainty satisfying

$$0 < \beta_{\min} \equiv \frac{g_{\min}}{g_m} \leqslant \Delta g \leqslant \frac{g_{\max}}{g_m} \equiv \beta_{\max}. \tag{5}$$

To deal with these uncertainties, traditional robust designs or adaptive strategies are not applicable. In Section 4, the function approximation technique is employed to represent $f(\mathbf{x}, t)$ as a finite combination of basis functions, and an adaptive sliding controller is designed to have desired performance. Besides, it should be noted that in addition to Eq. (3), there is a second-order internal dynamics [16] to be considered. By setting $y = \dot{y} = 0$, the zero dynamics [16] with zero road input $z = 0$ can be derived as

$$\dot{x}_3 = x_4, \tag{6a}$$

$$\dot{x}_4 = \frac{1}{m_u}[-k_t x_3 - c_t x_4 - \Delta m_s(t)g]. \tag{6b}$$

Fortunately, Eq. (6) is just the dynamics of the unsprung mass whose input is the bounded variation of the car-body load, and it is an ISS second-order system [17].

3. Review of function approximation technique

In this section some basic notions of function approximation with orthonormal functions are reviewed [18]. A set of real-valued functions $\{z_i(t)\}$ defined over some interval $[t_1, t_2]$ is said to form an *orthogonal set* on that interval if

$$\int_{t_1}^{t_2} z_i(t)z_j(t)\,\mathrm{d}t \begin{cases} = 0, & i \neq j, \\ \neq 0, & i = j. \end{cases} \tag{7}$$

An orthogonal set $\{z_i(t)\}$on $[t_1, t_2]$ having the property $\int_{t_1}^{t_2} z_i^2(t)\,\mathrm{d}t = 1$ for all i is called an *orthonormal set* on $[t_1, t_2]$. The set of real-valued functions $\{z_i(t)\}$ defined over some interval $[t_1, t_2]$ is orthogonal with respect to the weight function $p(t)$ on that interval if

$$\int_{t_1}^{t_2} p(t)z_i(t)z_j(t)\,\mathrm{d}t \begin{cases} = 0, & i \neq j, \\ \neq 0, & i = j. \end{cases} \tag{8}$$

Any set of functions orthogonal with respect to a weight function $p(t)$ can be converted into a set of functions orthogonal to 1 simply by multiplying each member of the set by $\sqrt{p(t)}$ if $p(t) > 0$ on that interval. For any set of orthonormal functions $\{z_i(t)\}$ on $[t_1, t_2]$, an arbitrary function $f(t)$ can be represented in terms of $z_i(t)$ by a series

$$f(t) = w_1 z_1(t) + w_2 z_2(t) + \cdots + w_n z_n(t) + \cdots. \tag{9}$$

This series is called a *generalized Fourier series* of $f(t)$ and its coefficients are Fourier coefficients of $f(t)$ with respect to $\{z_i(t)\}$. Multiplying $z_n(t)$, integrating over the interval $[t_1, t_2]$ and using the orthogonality property, the series becomes

$$\int_{t_1}^{t_2} f(t)z_n(t)\mathrm{d}t = w_n \int_{t_1}^{t_2} z_n^2(t)\mathrm{d}t. \tag{10}$$

Hence, the coefficient w_n can be obtained from the quotient

$$w_n = \frac{\int_{t_1}^{t_2} f(t)z_n(t)\,\mathrm{d}t}{\int_{t_1}^{t_2} z_n^2(t)\,\mathrm{d}t}. \tag{11}$$

It should be noted that although the orthogonality property can be used to determine all coefficients in (9), it is not sufficient to conclude convergence of the series. To guarantee convergence of the approximating series, the orthogonal set should be complete. An orthogonal set $\{z_i(t)\}$ on $[t_1, t_2]$ is said to be *complete* if the relation $\int_{t_1}^{t_2} h(t)z_i(t)\,\mathrm{d}t = 0$ can hold for all values of i only if $h(t)$ have non-zero values in a measure zero set in $[t_1, t_2]$. Here, $h(t)$ is called a *null* function on $[t_1, t_2]$ satisfying $\int_{t_1}^{t_2} h^2(t)\,\mathrm{d}t = 0$. It is easy to prove that if $\{z_i(t)\}$ is a complete orthonormal set on $[t_1, t_2]$ and the expansion $w_1 z_1(t) + w_2 z_2(t) + \cdots + w_n z_n(t) + \cdots$ of $f(t)$ converges and can be integrated term by term, then the sum of the series differs from $f(t)$ by at most a null function, and the series converges in the sense of mean square as

$$\lim_{n\to\infty} \int_{t_1}^{t_2} \left| f(t) - \sum_{i=1}^{n} w_i z_i(t) \right|^2 \mathrm{d}t = 0, \tag{12}$$

where the positive integer n is the number of terms used in approximation. This implies that any function $f(t)$ in the current Hilbert space can be approximated to arbitrarily prescribed accuracy by finite linear combinations of the orthonormal basis $\{z_i(t)\}$ as

$$f(t) = \sum_{i=1}^{n} w_i z_i(t) + \varepsilon, \tag{13}$$

where $\varepsilon = \sum_{i=n+1}^{\infty} w_i z_i(t)$ is the approximation error. Rewriting (13) into vector form, we have

$$f(t) = \mathbf{w}^{\mathrm{T}}\mathbf{z} + \varepsilon, \tag{14}$$

where $\mathbf{z}(t) = [\, z_1(t) \quad z_2(t) \quad \cdots \quad z_n(t)\,]^{\mathrm{T}}$ is the basis function vector and $\mathbf{w} = [\, w_1 \quad w_2 \quad \cdots \quad w_n\,]^{\mathrm{T}}$ is the time-invariant coefficient vector. An excellent property of (14) is its linear parameterization of the time-varying function $f(t)$ into the multiplication of $\mathbf{z}(t)$ and $\mathbf{w}$ with prescribed accuracy ε. If sufficient numbers of basis functions are used, then the approximation error can be further neglected, i.e., $\varepsilon \approx 0$. In Section 4, Eq. (14) is applied to represent the time-varying parameter in the system dynamic equation, where the time-varying vector $\mathbf{z}(t)$ is known while $\mathbf{w}$ is an unknown constant vector. With this approximation, the unknown time-varying function is replaced by a set of unknown constants; therefore, a proper Lyapunov function can be selected to find update laws for these unknown constants.

4. Controller design

In this section we would like to derive an adaptive sliding controller for the quarter-car suspension system, and the closed-loop stability is proved using the Lyapunov direct method.

Define a sliding surface

$$s = \dot{y} + \lambda y = 0, \tag{15}$$

where $\lambda > 0$ is the convergent rate of y on the sliding surface. The dynamics of the error signal s is calculated as

$$\dot{s} = \ddot{y} + \lambda \dot{y}. \tag{16}$$

Substituting (3) into (16), we have

$$\begin{aligned} \dot{s} &= f(\mathbf{x}, t) + g(t)u + \lambda \dot{y} \\ &= f(\mathbf{x}, t) + g_m(\Delta g - 1)u + g_m u + \lambda \dot{y}. \end{aligned} \tag{17}$$

Let us design the control u as

$$u = \frac{1}{g_m}[-\hat{f} - \lambda \dot{y} - \eta_1 s - \eta_2 \operatorname{sgn}(s)], \tag{18}$$

where $\hat{f}$ is the estimate of $f(\mathbf{x}, t)$. The constants $\eta_1 > 0$ and $\eta_2 > 0$ are parameters to be selected. Then Eq. (17) can be rewritten as

$$\dot{s} = -\eta_1 s - \eta_2 \operatorname{sgn}(s) + (f - \hat{f}) + g_m(\Delta g - 1)u. \tag{19}$$

Here, we would like to use the function approximation technique defined in (14) to represent f and $\hat{f}$ as

$$f = \mathbf{w}_f^{\mathrm{T}} \mathbf{z}_f + \varepsilon_f, \tag{20a}$$

$$\hat{f} = \hat{\mathbf{w}}_f^{\mathrm{T}} \mathbf{z}_f, \tag{20b}$$

where $\mathbf{w}_f, \hat{\mathbf{w}}_f \in \Re^n$ are weighting vectors, $\mathbf{z}_f \in \Re^n$ is the vector of basis functions and ε_f is the approximation error. Hence (19) becomes

$$\dot{s} = -\eta_1 s - \eta_2 \operatorname{sgn}(s) + \tilde{\mathbf{w}}_f^{\mathrm{T}} \mathbf{z}_f + g_m(\Delta g - 1)u + \varepsilon_f, \tag{21}$$

where $\tilde{\mathbf{w}}_f = \mathbf{w}_f - \hat{\mathbf{w}}_f$. It is observed that the error dynamics is driven by parameter errors. To find update laws for $\hat{\mathbf{w}}_f$ and to prove stability of the closed-loop system, a Lyapunov function candidate is designed as

$$V = \tfrac{1}{2}s^2 + \tfrac{1}{2}\tilde{\mathbf{w}}_f^{\mathrm{T}} \mathbf{Q}_f \tilde{\mathbf{w}}_f, \tag{22}$$

where $\mathbf{Q}_f \in \Re^{n\times n}$ is positive definite and symmetric. Taking the time derivative of V along the trajectory of (21), we have

$$\dot{V} = -\eta_1 s^2 - \eta_2 |s| + \tilde{\mathbf{w}}_f^{\mathrm{T}}(\mathbf{z}_f s - \mathbf{Q}_f \dot{\hat{\mathbf{w}}}_f) + g_m(\Delta g - 1)su + \varepsilon_f s. \tag{23}$$

If we select

$$\dot{\hat{\mathbf{w}}}_f = \mathbf{Q}_f^{-1} \mathbf{z}_f s, \tag{24}$$

$$\eta_2 = g_m(\beta_{\max} + 1)u_{\max}, \tag{25}$$

then (23) becomes

$$\dot{V} \leqslant (-\eta_1 |s| + |\varepsilon_f|)|s|. \tag{26}$$

Therefore, if a proper $\eta_1 > 0$ and a suitable set of basis functions are chosen, then $\dot{V} \leqslant 0$ whenever $s \in \{\sigma \,|\, |\sigma| > |\varepsilon_f|/\eta_1\}$. This implies that the output error s is uniformly bounded.

Remark 1. If a sufficient number of basis functions are used so that $\varepsilon_f \approx 0$, then (26) implies $s \in L_\infty \cap L_2$, and $\tilde{\mathbf{w}}_f \in L_\infty$. In addition, Eq. (21) implies $\dot{s} \in L_\infty$. Hence, the asymptotic convergence of output error s can be concluded using the Barbalat's lemma [19].

Remark 2. If ε_f cannot be ignored but its variation bound can be estimated [12–14], i.e. there exists a positive constant $\delta > 0$ such that $|\varepsilon_f| \leqslant \delta$, then the controller parameter η_2 can be modified as

$$\eta_2 = g_m(\beta_{\max} + 1)u_{\max} + \delta$$

With the selection of update laws in (24), $\dot{V}$ can be derived to be

$$\dot{V} \leqslant -\eta_1 s^2 + |\varepsilon_f||s| - \delta |s| \leqslant 0$$

Therefore, we may further conclude asymptotic convergence of s according to the Barbalat's lemma.

In this paper, we only investigate the case when sufficient numbers of basis functions are used, and hence no approximation error is considered. Under this condition, the output error is proved to have asymptotically convergent performance. This further implies that the sprung mass displacement converges as desired. It is observed from (18) and (24) that to realize the controller we do not need to feedback the position or velocity of the unsprung mass and this much simplifies the hardware structure in implementations.

5. Simulations

To verify the effectiveness of the proposed method, computer simulations are performed with the following plant parameters: $m_s(t) = 290 + 60 \sin t\,(\mathrm{kg})$, $m_u = 59\,(\mathrm{kg})$, $k_{1s} = 14500\,(\mathrm{N/m})$, $k_{2s} = 160000\,(\mathrm{N/m^3})$, $k_t = 190000\,(\mathrm{N/m})$, $c_s(t) = 900 + 200e^{-0.1t}\,(\mathrm{N/m/s})$, $c_t = 170\,(\mathrm{N/m/s})$, and $u_{\max} = 3000\,(\mathrm{N})$. Assuming that $m_{sm} = 290(\mathrm{kg})$ and the bound of $\Delta m_s(t)$ is 70 (kg), the parameters g_m, $\beta_{\max}$ and $\beta_{\min}$ in (5) are calculated as $g_m = 0.00356$, $\beta_{\max} = 1.278$ and $\beta_{\min} = 0.781$, respectively. The static deflections of the suspension and tire can be computed as $\delta_0 = -0.155\,(\mathrm{m})$ and $x_{ur} = -0.018\,(\mathrm{m})$, respectively, due to the car-body nominal load $m_{sm}g = 2845\,(\mathrm{N})$. The controller parameters are $\eta_1 = 1$ and $\lambda = 1$, and the value of η_2 is calculated from Eq. (25) to be 20.274. To avoid chattering in the control action, the $\mathrm{sgn}(s)$ function in (18) is replaced by the saturation function $\mathrm{sat}(s/\phi)$ with the boundary layer thickness $\phi = 0.05$. The first nine terms of the Fourier series [18] are employed as the basis functions. The matrix $\mathbf{Q}_f \in \Re^{9\times 9}$ is chosen as a diagonal matrix and all of its diagonal elements are 0.002. The initial weighting matrix is set to be $\hat{\mathbf{w}}_f = [\,0.01 \quad 0.01 \quad \cdots \quad 0.01\,]^{\mathrm{T}}$, and the system is initially with $\mathbf{x}(0) = [\,0 \quad 0 \quad 0 \quad 0\,]^{\mathrm{T}}$. We would like the sprung mass to stay at its reference position x_{sr} even with the road disturbance. The uneven terrain is composed of a positive bump followed by a negative bump. Sinusoidal disturbances are also superimposed on the road profile to simulate the rough road surface. Therefore, the terrain disturbance input has the form:

$$z_r(t) = \begin{cases} -0.0592(t-3.5)^3 + 0.1332(t-3.5)^2 + d(t) & \text{for } t \in [3.5, 5), \\ 0.0592(t-6.5)^3 + 0.1332(t-6.5)^2 + d(t) & \text{for } t \in [5, 6.5), \\ 0.0592(t-8.5)^3 - 0.1332(t-8.5)^2 + d(t) & \text{for } t \in [8.5, 10), \\ -0.0592(t-11.5)^3 - 0.1332(t-11.5)^2 + d(t) & \text{for } t \in [10, 11.5), \\ d(t) & \text{else.} \end{cases} \tag{27}$$

where $d(t) = 0.002 \sin 2\pi t + 0.002 \sin 7.5\pi t$ is the sinusoidal disturbance.

Case 1: In this case, the sprung mass is assumed to be constant with $m_s = 290\,(\mathrm{kg})$. Simulation results are shown in Figs. 3–9. In Fig. 3, it can be seen that the sprung mass regulation is dramatically improved with the proposed controller compared with the passive counterpart. Fig. 4 is the PSD plot of the car-body acceleration, which shows significant improvement of the car-body vibration in the entire spectrum. Fig. 5 is the suspension deflection. Fig. 6 is the curve of the tire deflection where the negative deflection means that the tire keep contacting with the ground. It can be observed that the tire with the proposed method is able to keep on the ground well at all times and hence the road holding ability will not be deteriorated. Fig. 7 presents the function

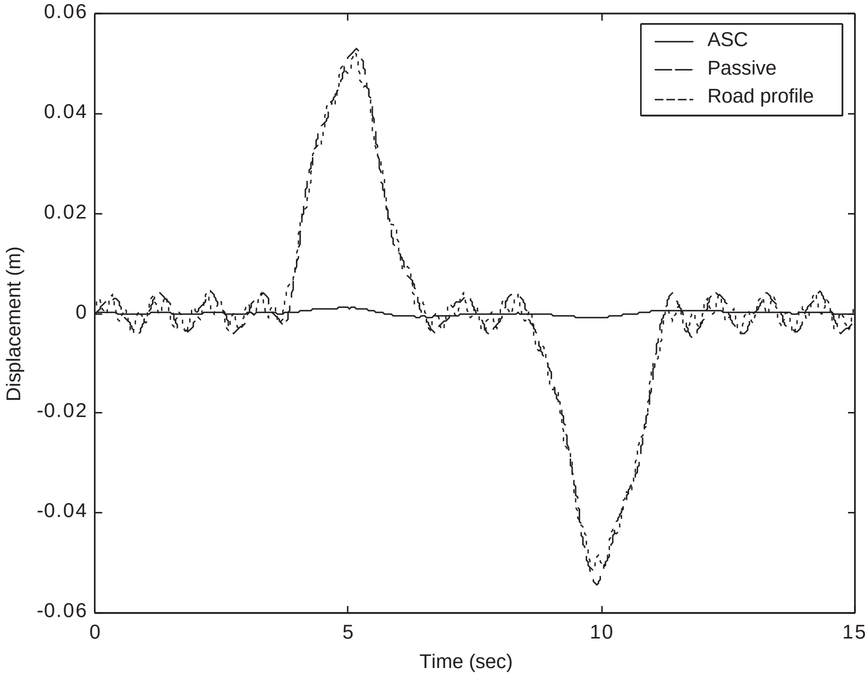

Fig. 3. Sprung mass displacement (x_1) in Case 1.

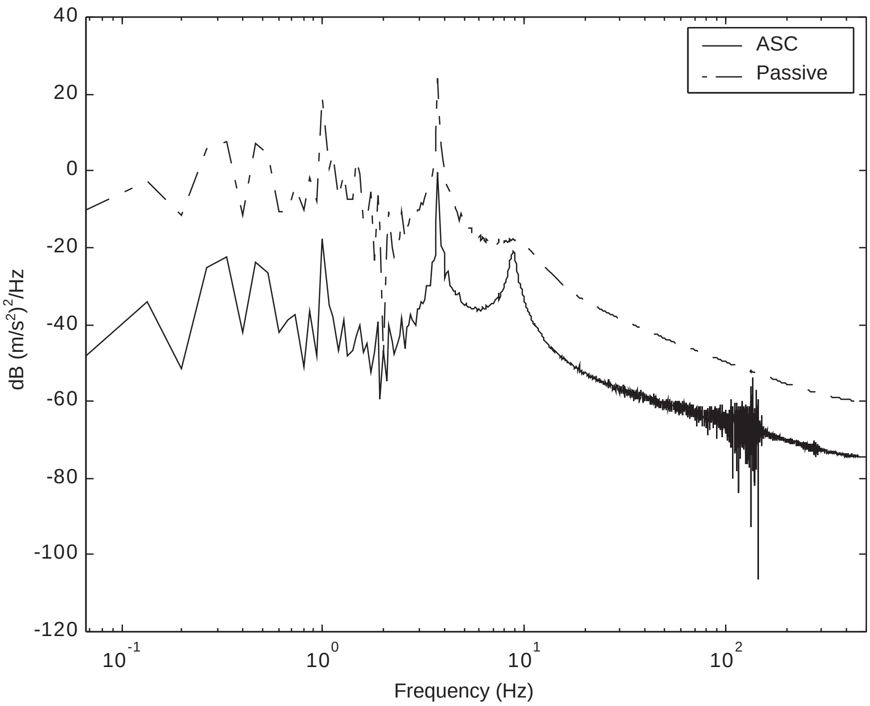

Fig. 4. PSD of sprung mass acceleration in Case 1.

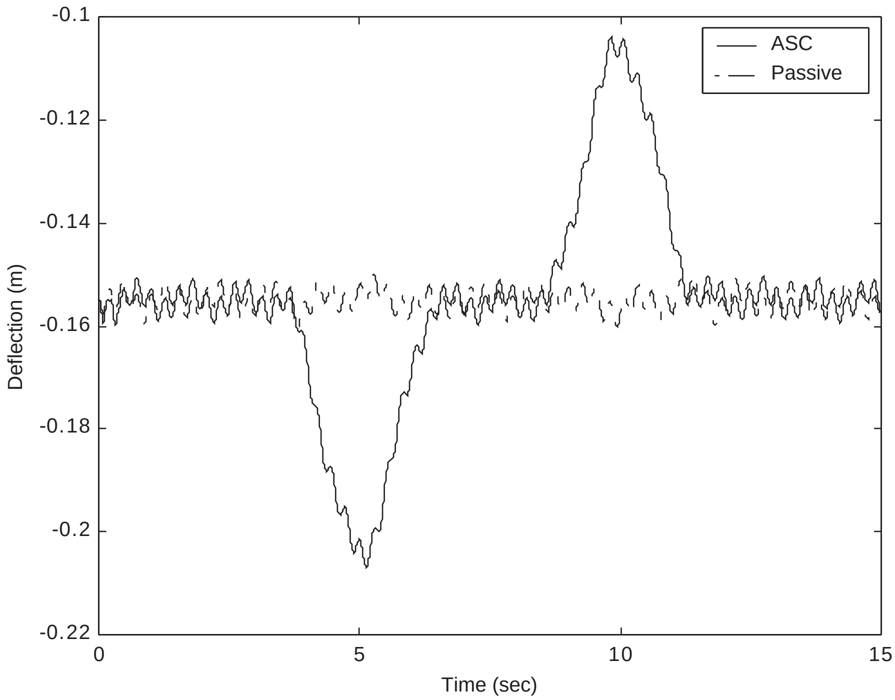

Fig. 5. Suspension deflection ($x_s - x_u$) in Case 1.

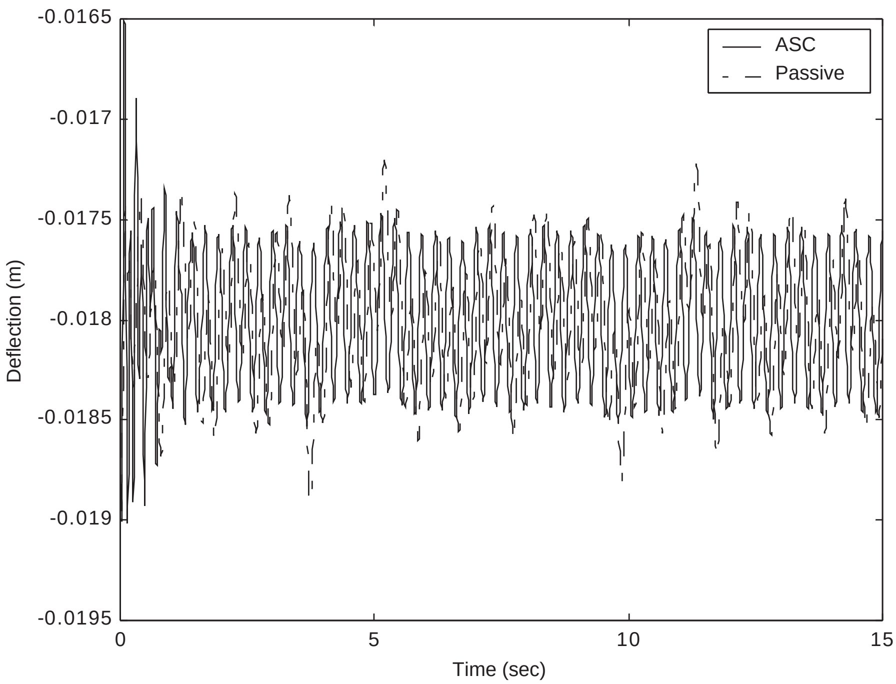

Fig. 6. Tire deflection ($x_u - z$) in Case 1.

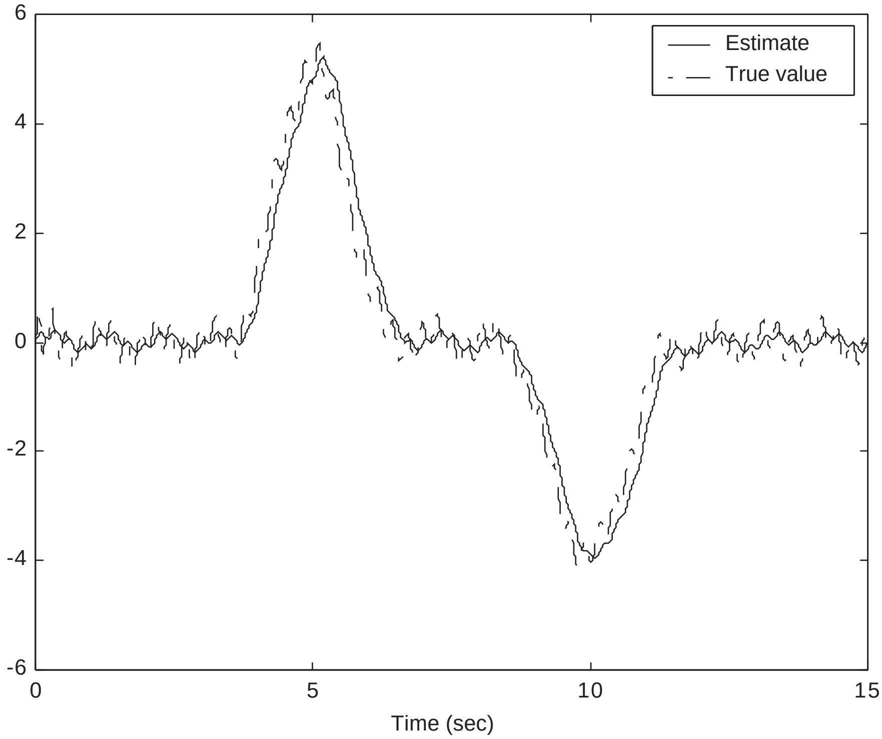

Fig. 7. Approximation of $f(\mathbf{x},t)$ in Case 1.

approximation performance for the unknown time-varying function $f(\mathbf{x}, t)$ and Fig. 8 gives the time history of the control force.

Case 2: The sprung mass is considered as a fast time-varying function $m_s(t) = 290 + 60 \sin t\,(\mathrm{kg})$ and simulation results are shown in Figs. 9–14 with satisfactory performance. In Fig. 9, it is observed that due to the asymmetric property of the nonlinear spring and time-varying loading, the passive system exhibits a larger variation in sprung mass displacement than the road profile. On the contrary, both the displacement and acceleration of the sprung mass with the proposed controller shown, respectively, in Figs. 9 and 10 are effectively regulated under severe loading conditions regardless of the system uncertainties. Reasonable control efforts can also be seen in Fig. 14 to show the feasibility of the proposed method.

6. Conclusions

This paper proposes an adaptive sliding controller for a non-autonomous quarter-car suspension system. The system model is firstly represented with respect to the static positions under the nominal car-body load. In order to cope with the system nonlinearities and uncertainties, the function approximation technique is applied. Then the control rule and the update laws are designed to guarantee the closed-loop stability. Although the system contains

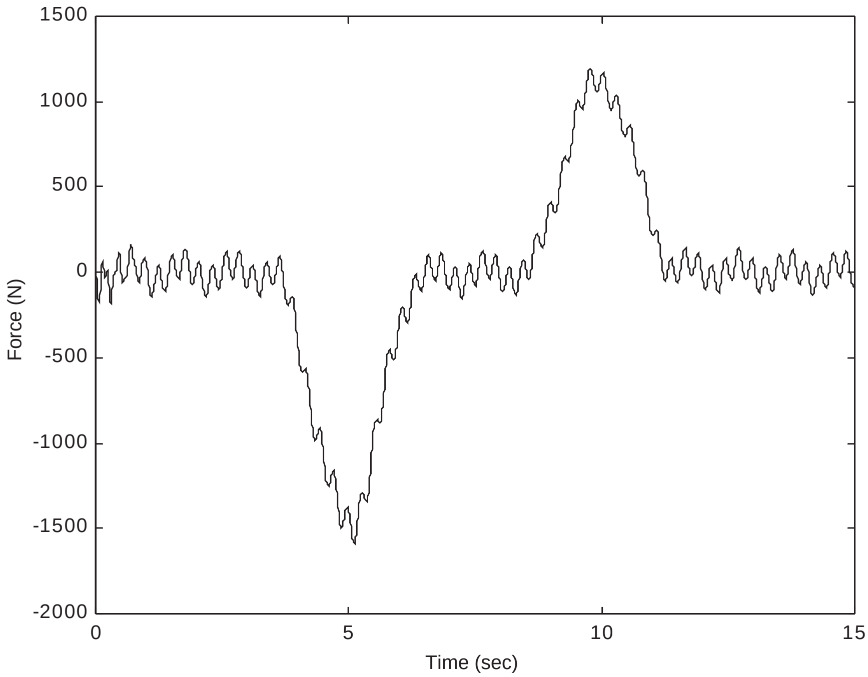

Fig. 8. Control force in Case 1.

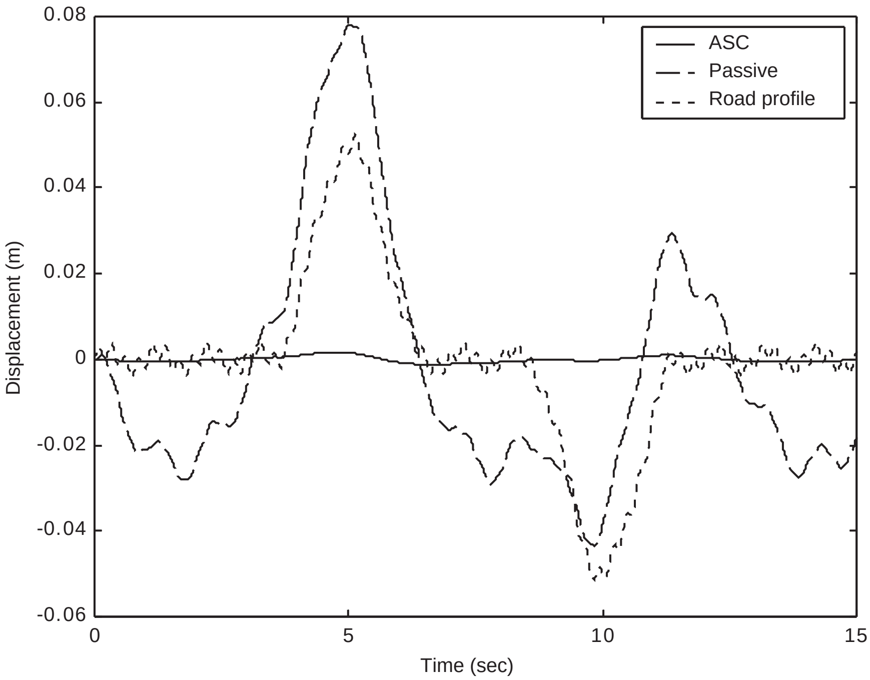

Fig. 9. Sprung mass displacement (x_1) in Case 2.

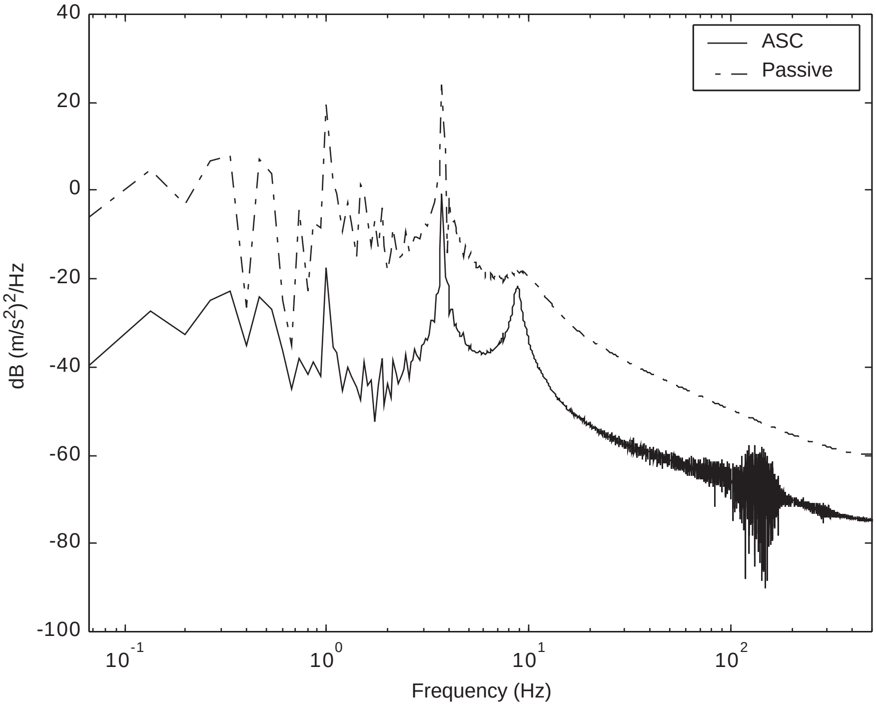

Fig. 10. PSD of sprung mass acceleration in Case 2.

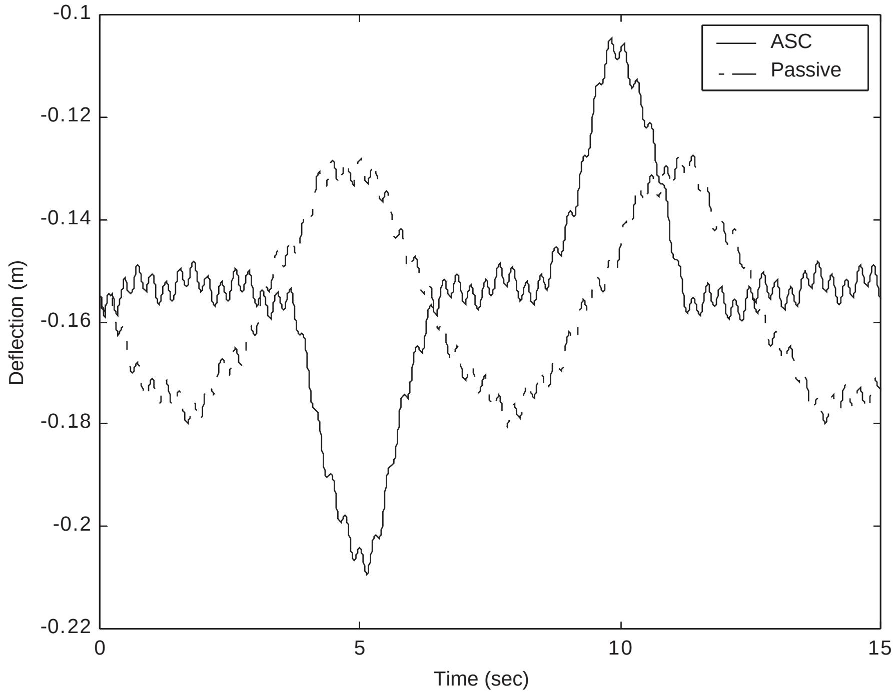

Fig. 11. Suspension deflection $(x_s - x_u)$ in Case 2.

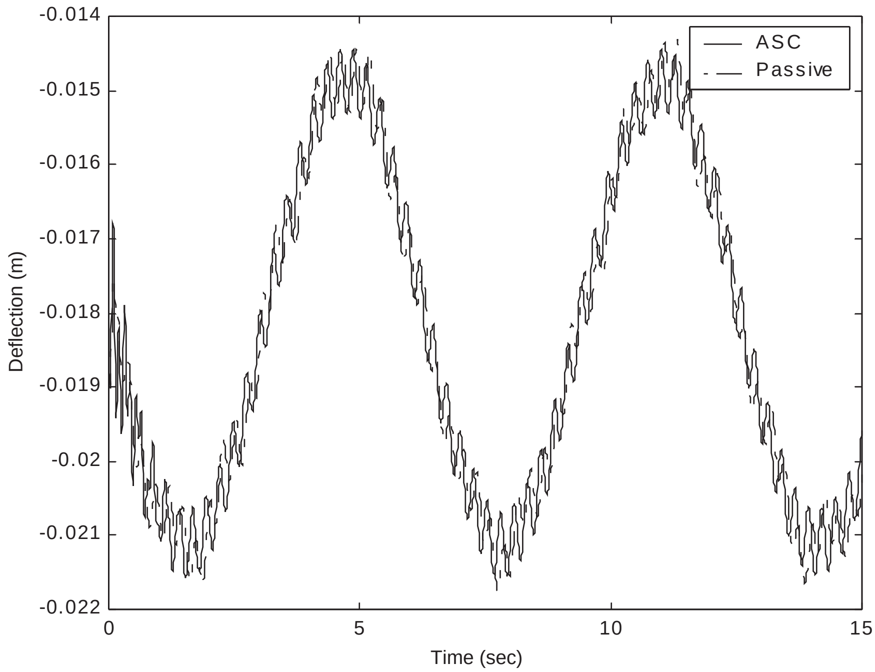

Fig. 12. Tire deflection ($x_u - z$) in Case 2.

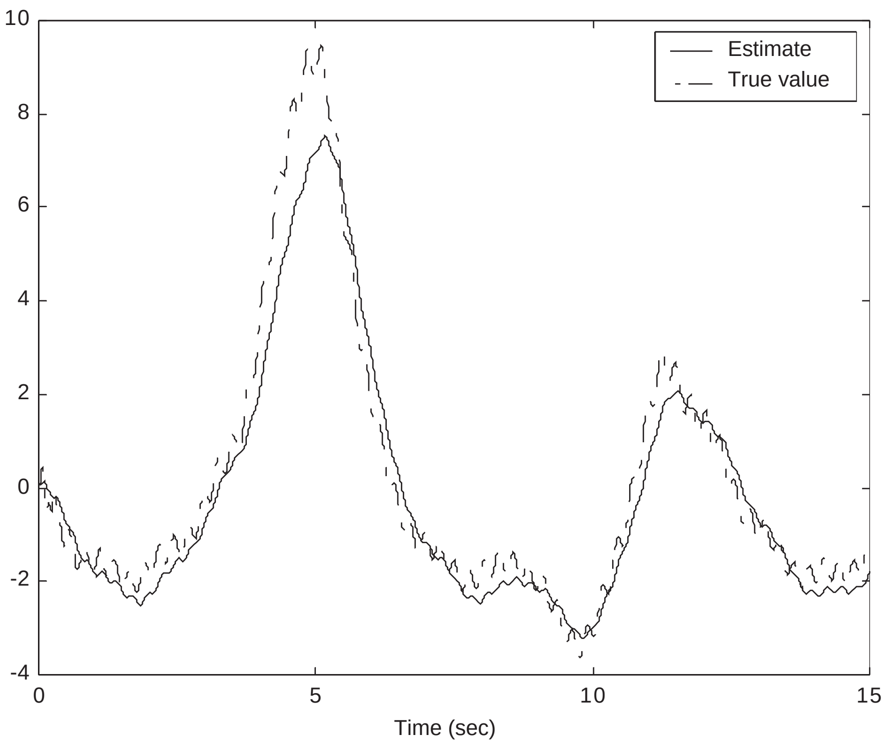

Fig. 13. Approximation of $f(\mathbf{x},t)$ in Case 2.

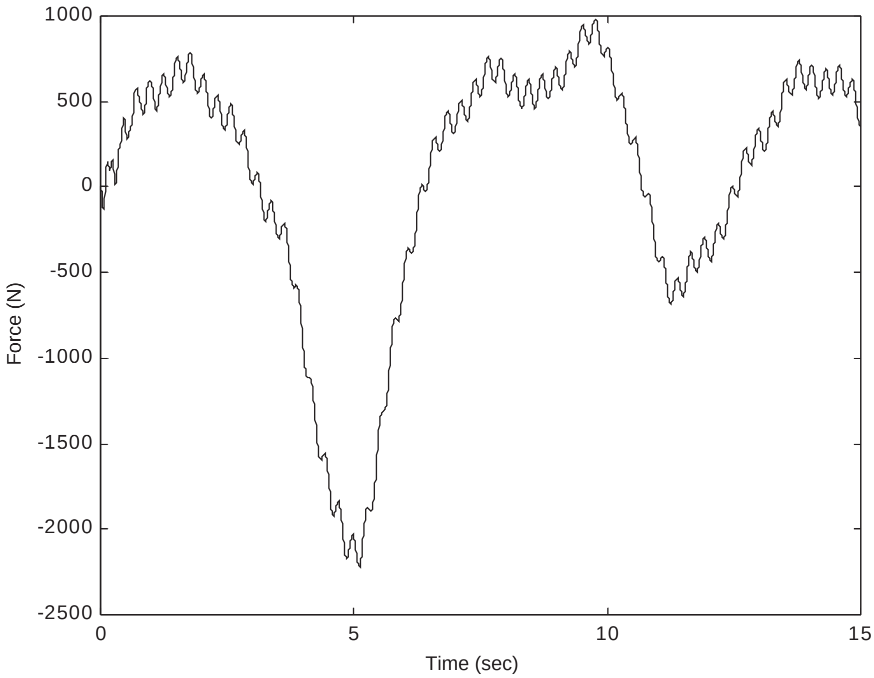

Fig. 14. Control force in Case 2.

time-varying uncertainties, the proposed controller gives significant performance improvement compared with the pure passive design from the viewpoint of ride comfort. Besides, the controller can be realized with only position and velocity feedback of the car body, and this implies great simplification in hardware implementation. Reasonable control activity shows its feasibility of realization using available hydraulic components.

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